

FENNM: A Hybrid Variational Framework for Robust Structural Modelling and Inverse Problem Solving

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Many of the important challenges in the modelling of composite materials and their processing can be formulated as inverse problems: identifying the rheological properties of a resin from observations on how it flows into a real part, identifying the multi-scale properties of a composite part from micro- or macro-scale observations, or tuning the fracture mechanics of a model based on experimental observations. While Physics-Informed Neural Networks (PINNs) offer a promising framework for combining experimental data with mathematical models to solve these inverse problems, they often lack robustness in practical engineering settings. Standard PINNs are sensitive to initialization and typically require extensive problem-specific tuning to balance competing loss terms, which can hinder convergence for stiff or highly nonlinear systems [1, 2].

To address these limitations, this research introduces the Finite Element Neural Network Method (FENNM), a hybrid solver that integrates the data-driven flexibility of deep learning with the rigorous formalism of the finite element method (FEM). FENNM is developed within the Petrov-Galerkin framework, where a neural network generates the global nonlinear space of trial solutions while the test functions are selected from the nonvanishing Lagrange shape function space.

Unlike previous variational PINN variants that utilize vanishing test functions—thereby losing critical interface information—FENNM explicitly incorporates flux terms at element boundaries. This architectural choice allows natural boundary conditions and point loads to be embedded directly into a single residual loss function, significantly enhancing training stability. To maintain computational efficiency, we utilize tensor product operations to evaluate Gauss quadrature integration across the domain as detailed.

The method's efficacy is demonstrated through case studies pivotal to composite structural modelling. We showcase FENNM's ability to solve inverse and parametric identification problems, identifying unknown physical properties—such as surface absorptivity in a thermal panel model—using as little as a single data point. Furthermore, FENNM is shown to function effectively on irregular meshes generated by standard industrial tools like Gmsh. By bridging the gap between machine learning and traditional numerical methods, FENNM provides a robust framework for the next generation of structural modelling tools in the aerospace and construction industries.

References:

[1] Abda, M., Hamedi, M., Piollet, E., Blake, C., Gosselin, F.P. “The Finite Element Neural Network Method: One Dimensional Study” *Next Research*, 2, 4, 100885 (2025)

[2] Abda, M., Berthet, L., Hamedi, M., Hibatullah, D., Piollet, E., Blake, C., Blais, B., Gosselin, F.P. “The Finite Element Neural Network Method: Leveraging Non-Vanishing Shape Functions in Space-Time-Parameter Framework”, *Research Square Preprint* [<https://doi.org/10.21203/rs.3.rs-7702801/v1>]